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Distribution Line Models

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1. Overview

- In a distribution system, power is transmitted from a substation to loads through three-phase, two-phase, or single-phase lines.
- These lines can be either overhead or underground.
- In order to accurately analyze the behavior of distribution systems, it is necessary to develop models that represent the characteristics of these lines.
- These models can be used to simulate the behavior of the lines under various operating conditions and to evaluate the performance of the distribution system.

2. Conductor Types and Sizes

2.1 Sizes

- These numbers start at 0000 (four zeros or four-) to higher numbers. The conductors from 0000 to 0 are designated by the number followed by slash and a zero. For example, 0000 is called 4/0, and 0 is called 1/0 in utility jargon.
- Conductors smaller than these go from #1 to #6. Conductors smaller than #6 are too small for distribution systems.
- AWG sizes are also used for wires used in homes. Typically, house wiring uses #12 copper conductors. Extension cords use #14 to #18 copper conductors.

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2.2 Overhead Feeders

- Primary side of the Distribution Systems most often use aluminum conductor steel-reinforced (ACSR) conductors.
- ACSR conductors have a steel core surrounded by aluminum strands.
- Overhead cables for secondary service drop are all aluminum.
- Live conductors are covered with insulation, but neutral is bare.

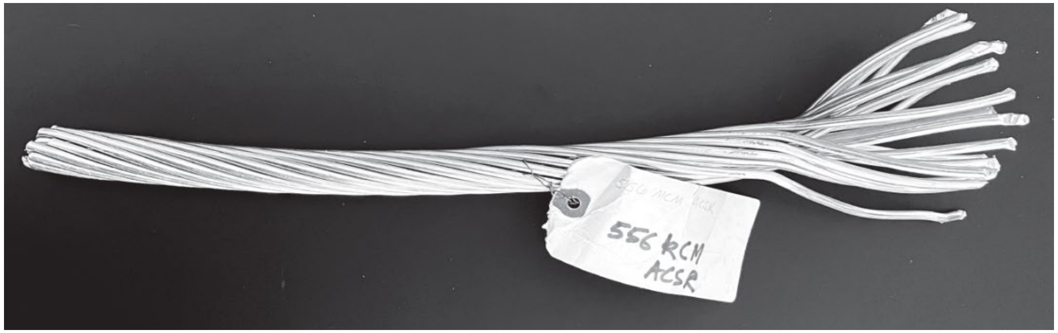


Fig (a): 556 kcmil ACSR overhead conductor

2.2 Overhead Feeders

- Figure (b) shows #2 Triplex cable used for single-phase residential service.
- For three-phase service, quadraplex conductors are used.

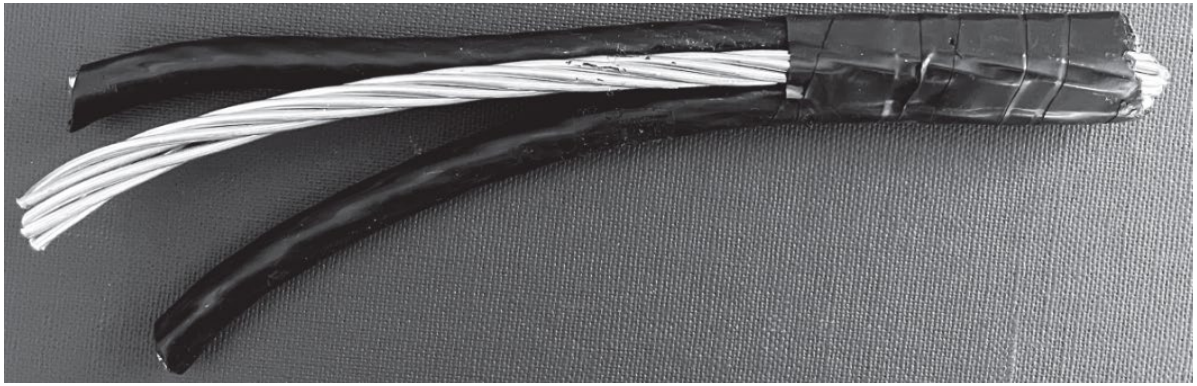


Fig (b): #2 Triplex cable

2.3 Underground Feeders

- Underground feeders are built with either copper or aluminum cables. The cables used for the primary side of the distribution system have a layer of insulation and an outer jacket.

- Figure on the right, shows an example of an insulated 15 kV class copper cable. They are laid in ducts or directly buried. Triplex and quadraplex cables suitable for direct burial in ground are used for service drops.

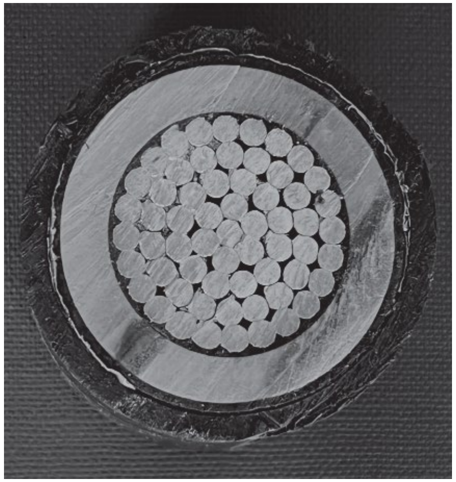


Fig : Cross section of a 15kV class insulated copper cable

2.4 Conductor Data

- Manufacturers of conductors provide detailed data based on the construction geometry.
- In addition, electrical properties, such as resistance, are provided.
- Also provided is a quantity called Geometric Mean Radius (GMR), which is used in the computation of line parameters.

2.4 Conductor Data

- The GMR of a conductor made with N strands twisted together is given by:

$$\text{GMR} = \sqrt[N^2]{\prod_{i} \prod_{k} d_{ik}} \quad ; i, k = 1 \text{ to } N$$

Where, d_{ik} is the distance of each strand from every other strand in the ensemble, and d_{ii} is the radius of the individual strand.

<u>Note:</u> The radius is multiplied by a factor of $e^{-1/4}$ to account for internal magnetic flux while computing the inductance of conductors.

3. Generalized Carson's Model

- Carson did seminal work in the 1920s on modeling of overhead lines.
- All the models used today for transmission and distribution feeders are based on his work.
- To understand his work, consider an overhead line shown in the Figure.

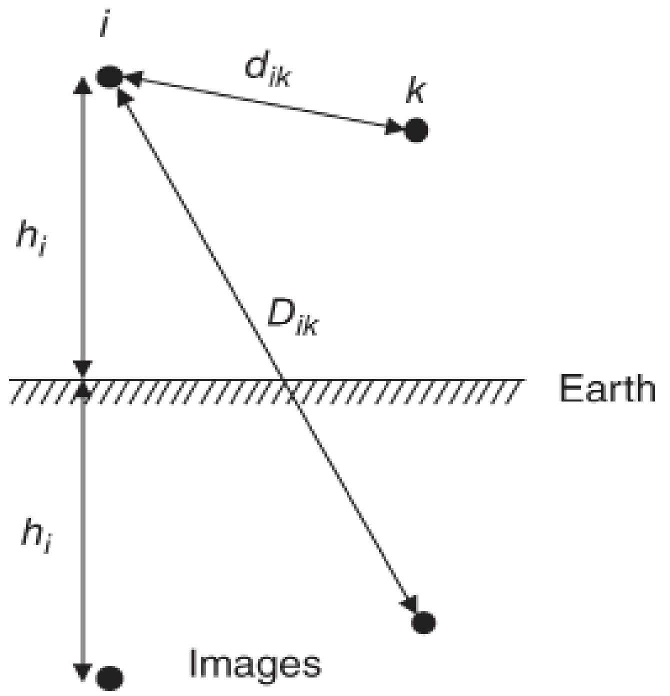


Fig: Overhead lines and their images below earth

3. Generalized Carson's Model

- It is assumed that the lines are long enough to neglect end effects.
- The earth is assumed to have uniform conductivity and is bounded by a flat plane of infinite extent, with conductors parallel to this plane.
- In practice, the conductors may sag. To account for this, the average height above ground is calculated by adding the height at the midpoint and one-third of the sag.
- The figure also shows image conductors, which are located at depths below the earth equal to the heights of the conductors above the earth.

3. Generalized Carson's Model

- Self-inductance of conductor i :

$$Z_{iig} = (R_{ii} + \Delta R_{iig}) + j\left(2\omega \cdot 10^{-4} \ln \frac{2h_i}{\mathrm{GMR}_i} + \Delta X_{iig}\right) \Omega/\mathrm{km}$$

- Mutual inductance between conductors i and k :

$$Z_{ikg} = Z_{kig} = \Delta R_{ikg} + j\left(2\omega \cdot 10^{-4} \ln \frac{D_{ik}}{d_{ik}} + \Delta X_{ikg}\right) \Omega/\mathrm{km}$$

R_{ii}, resistance of conductor i in Ω/km;
h_i, height above ground of conductor i;
D_{ik}, distance between conductor i and image of conductor k;
d_{ik}, distance between conductors i and k;
GMR_i, geometric mean radius of conductor i;
$\omega = (2\pi f)$ with f = frequency in Hertz; ΔR_{ii} , ΔX_{ii} , ΔR_{ik}, ΔX_{ik} are corrections to the respective terms for the earth return effect.

3. Generalized Carson's Model

- The final equations that are widely accepted and prevalent for computing the self and mutual impedances of overhead conductors including the ground effect is,
- Self-impedance of conductor i :

$$Z_{iig} = (R_{ii} + 0.00159f) + j\left(0.004657f\log\frac{2160\sqrt{\frac{\rho}{f}}}{\mathrm{GMR}_i}\right)\Omega/\mathrm{mile}$$

- Mutual impedance between conductors i and k :

$$Z_{ikg} = Z_{kig} = 0.00159f + j\left(0.004657f\log\frac{2160\sqrt{\frac{\rho}{f}}}{d_{ik}}\right)\Omega/\mathrm{mile}$$

3. Generalized Carson's Model

where

R_{ii}, resistance of conductor i in Ω/ mile;
d_{ik}, distance between conductors i and k in feet;
GMR_i, geometric mean radius of conductor i in feet;
f, frequency in Hertz;
ρ, resistivity of earth in ohm-meter (usually 100Ω - m is used as the typical value).

Note the mixed nature of units in these equations. While feet and miles are used for all the quantities, resistivity of earth is given in ohm-meter. This is a leftover from the legacy work on this subject and continues to be used with mixed units.

4. Series Impedance Models of **Overhead** Lines

4.1 Three-phase Line

- A typical configuration of a three-phase distribution feeder is shown in the Figure.
- The formulas presented in the previous section is used to compute the series impedance matrix of this line based on the assumptions:
  - The neutral is grounded at multiple points including each pole.
  - The conductors are not transposed.

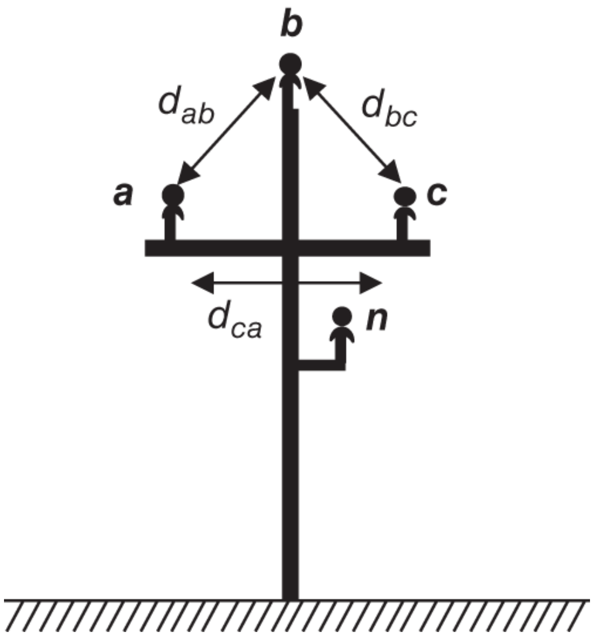


Fig: Typical three-phase overhead line configuration

4. Series Impedance Models of **Overhead** Lines

4.1 Three-phase Line

- Transposition is the practice of interchanging the position of conductors after a certain distance. Transposition is implemented in transmission systems, and it allows balancing the impedances of the three phases for a three-phase line.
- The shunt capacitance of most normal overhead distribution circuits is very low and makes no appreciable effect on the outcome of their steady-state performance.

4.1 Three-phase Line

- A feeder can be divided into sections, typically the portion between two interconnection points or nodes.
- Each section has an associated matrix describing the impedance, including self and mutual impedances of conductors in that section.
- Using Ohm's law, the impedance matrix is post multiplied by the current phasor vector to obtain the voltage drop per phase in the section between buses x and y:

4.1 Three-phase Line

$$\begin{bmatrix} V_{xy}^{a} \\ V_{xy}^{b} \\ V_{xy}^{c} \\ V_{xy}^{n} \end{bmatrix} = \begin{bmatrix} Z_{aag} & Z_{abg} & Z_{acg} & Z_{ang} \\ Z_{bag} & Z_{bbg} & Z_{bcg} & Z_{bng} \\ Z_{cag} & Z_{cbg} & Z_{ccg} & Z_{cng} \\ Z_{nag} & Z_{nbg} & Z_{ncg} & Z_{nng} \end{bmatrix} \begin{bmatrix} I_{xy}^{a} \\ I_{xy}^{b} \\ I_{xy}^{c} \\ I_{xy}^{n} \end{bmatrix}$$

Where,

$\boldsymbol{V}_{xy}^{i}$, phasor voltage drop across buses x and y on conductor $i, i = a, b, c, n$;

$\boldsymbol{I}_{xy}^{i}$, phasor current flowing in conductor i between buses x and $y, i = a, b, c, n$;

Z_{iig}, self-impedance of conductor i including the ground effect, $i = a, b, c, n$;

Z_{ikg}, mutual impedance between conductors i and k including the ground effect; $i, a, b, c, n; k = a, b, c, n; i \neq k$.

4.1 Three-phase Line

- From the equations of Generalized Carsen’s Model, the elements of the impedance matrix is calculated.

For example,

$$Z_{aag} = (R_{aa} + 0.00159f) + j\left(0.004657f\log\frac{2160\sqrt{\frac{\rho}{f}}}{\mathrm{GMR}_a}\right)\Omega/\mathrm{mile}$$

And,

$$Z_{abg} = Z_{bag} = 0.00159f + j\left(0.004657f\log\frac{2160\sqrt{\frac{\rho}{f}}}{d_{ab}}\right)\Omega/\mathrm{mile}$$

4.1 Three-phase Line

- The elements of the impedance matrix can be computed using appropriate equations.
- The system analysis is usually focused on the three phases, so the neutral can be removed using Kron's reduction.
  - This results in a 3x3 matrix from a 4x4 matrix.
  - The assumption is that the voltage drop across the neutral conductor is zero due to grounding at multiple locations.
  - most of the current will flow through the ground and only a very small amount will flow through the neutral.
- This gives the equation as,

4.1 Three-phase Line

$$V_{xy}^{n} = 0 = Z_{nag} I_{xy}^{a} + Z_{nbg} I_{xy}^{b} + Z_{nbg} I_{xy}^{c} + Z_{nng} I_{xy}^{n}$$

On solving, we get,

$$I_{xy}^{n} = -\frac{Z_{nag} I_{xy}^{a} + Z_{nbg} I_{xy}^{b} + Z_{nbg} I_{xy}^{c}}{Z_{nng}}$$

Substituting the value of I_{xy}^{n} in the equation of the impedance matrix, to compute V_{xy}^{a} yields

$$V_{xy}^{a} = Z_{aag} I_{xy}^{a} + Z_{abg} I_{xy}^{b} + Z_{acg} I_{xy}^{c}$$

$$-Z_{ang} \cdot \left(\frac{Z_{nag} I_{xy}^{a} + Z_{nbg} I_{xy}^{b} + Z_{nbg} I_{xy}^{c}}{Z_{nng}} \right)$$

4.1 Three-phase Line

Rearranging,

$$V_{xy}^{a} = \left(Z_{aag} - \frac{Z_{\text{ang}}\, Z_{nag}}{Z_{nng}}\right) I_{xy}^{a} + \left(Z_{abg} - \frac{Z_{\text{ang}}\, Z_{nbg}}{Z_{nng}}\right) I_{xy}^{b} + \left(Z_{acg} - \frac{Z_{ang} Z_{ncg}}{Z_{nng}}\right) I_{xy}^{c}$$

Similar equations can be obtained for $\boldsymbol{V}_{xy}^{b}$ and $\boldsymbol{V}_{xy}^{c}$.

Series Impedance Models of Overhead Lines

- Now, we can define the modified values of the self and mutual impedances

$$Z'_{iig} = \left(Z_{iig} - \frac{\left(Z_{ing}\right)^2}{Z_{nng}} \right); i = a, b, c$$

And,

$$Z'_{ikg} = \left(Z_{ikg} - \frac{Z_{ing} Z_{nkg}}{Z_{nng}} \right); i, k = a, b, c \text{ and } i \neq k$$

where Z'_{iig} represents self-impedance of conductor *i* including the ground and neutral current effects; Z'_{ikg} represents mutual impedance between phases *i* and *k* including the ground and neutral current effects.

Series Impedance Models of Overhead Lines

The previous derived 4×4 matrix can be rewritten in the reduced form to get a 3×3 matrix to get the phase domain model for series line parameters.

$$\begin{bmatrix} V_{xy}^{a} \\ V_{xy}^{b} \\ V_{xy}^{c} \end{bmatrix} = \begin{bmatrix} Z'_{aag} & Z'_{abg} & Z'_{acg} \\ Z'_{bag} & Z'_{bbg} & Z'_{bcg} \\ Z'_{\text{cag}} & Z'_{cbg} & Z'_{ccg} \end{bmatrix} \begin{bmatrix} I_{xy}^{a} \\ I_{xy}^{b} \\ I_{xy}^{c} \end{bmatrix}$$

Note : mutual impedance terms between any two conductors are the same due to symmetry.

For example, $\boldsymbol{Z}'_{abg} = \boldsymbol{Z}'_{\text{bag}}$ and so on for the other pairs of conductors.

4.2 Single- and Two-phase line modeling

- Single- and two-phase line models can be deduced logically using the same approach as described for the three-phase models. The nonzero values of impedances are essentially the same as in the three-phase case, but the entries for the phases that do not exist are zero.

4.2 Single- and Two-phase line modeling

- The impedance matrix for the single-phase line for phase a is:

$$\begin{bmatrix} \boldsymbol{Z}'_{aag} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- Similarly, the impedance matrix for a two-phase line with phases a and b is

$$\begin{bmatrix} Z'_{aag} & Z'_{abg} & 0 \\ Z'_{bag} & Z'_{bbg} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

***Note:** If the line has phases other than a and b, the matrix is developed accordingly.*

4.3 Three-phase Line Example

Consider a three-phase line with the configuration given in Figure 3.6. Consider the phase conductors to be 636 kcmil 54/7 (54 strands of aluminum and 7 strands of steel) ACSR, and the neutral conductor to be 2/0 ACSR. Consider earth resistivity of 100 Ω-m.

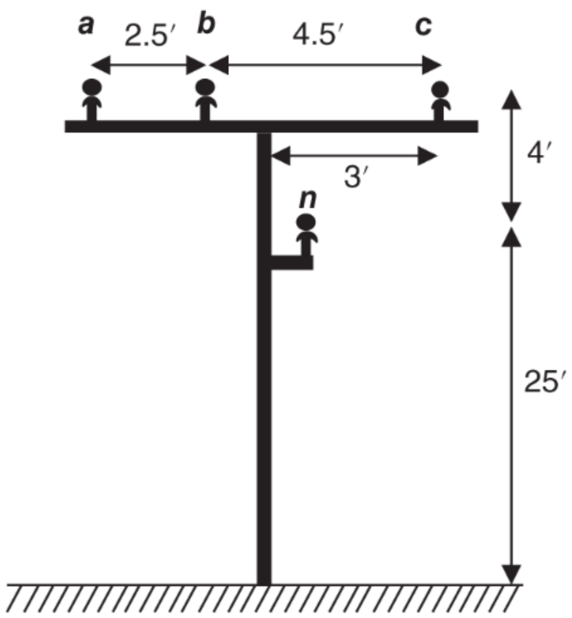


Fig: Three-phase overhead line configuration.

Solution

We can compute the distances between the conductors for the configuration:

$$D_{ab} = 2.5\text{ft}, D_{bc} = 4.5\text{ft}, D_{ca} = 7\text{ft}$$

and

$$D_{an} = \sqrt{4^2 + 4^2} = 5.657\text{ft}$$
$$D_{bn} = \sqrt{(1.5)^2 + 4^2} = 4.272\text{ft}$$
$$D_{cn} = \sqrt{3^2 + 4^2} = 5\text{ft}$$

Now, we use the conductor data sheet to find the relevant information.

$$\text{GMR} = 0.0329\text{ft}$$

Solution

Resistance has multiple values and different frequencies and temperatures. We select the value given for higher current at 60 Hz, or R = 0.1688 Ω/mile. Similarly, for the neutral conductor, GMR = 0.0051 ft and R = 0.706 Ω/mile. Note that we have selected the resistance at the lower current values for the neutral because in a three-phase line, neutral current is very small.

Solution

Next, we compute all the self and mutual impedances. For example,

$$Z_{aag} = (R_{ii} + 0.00159f) + j\left(0.004657f\log\frac{2160\sqrt{\frac{\ell}{f}}}{\mathrm{GMR}_i}\right)$$

$$= (0.1688 + 0.00159 \times 60) + j\left(0.004657 \times 60\log\frac{2160\sqrt{\frac{100}{60}}}{0.0329}\right)$$

$$= 0.2572 + j1.3770\ \Omega/\mathrm{mile}$$

$$Z_{abg} = Z_{bag} = 0.00159f + j\left(0.004657f\log\frac{2160\sqrt{\frac{\rho}{f}}}{d_{ab}}\right)$$

$$= 0.00159 \times 60 + j\left(0.004657 \times 60\log\frac{2160\sqrt{\frac{100}{60}}}{2.5}\right)$$

$$= 0.0954 + j0.8515\ \Omega/\mathrm{mile}$$

Solution

Repeating the same procedure gives the following impedance matrix for the line:

$$\begin{bmatrix} 0.2572 + j1.3770 & 0.0954 + j0.8515 & 0.0954 + j0.7265 & 0.0954 + j0.7523 \\ 0.0954 + j0.8515 & 0.2572 + j1.3770 & 0.0954 + j0.7801 & 0.0954 + j0.7865 \\ 0.0954 + j0.7265 & 0.0954 + j0.7801 & 0.2572 + j1.3770 & 0.0954 + j0.7674 \\ 0.0954 + j0.7523 & 0.0954 + j0.7865 & 0.0954 + j0.7674 & 0.8013 + j1.6032 \end{bmatrix} \Omega/\text{ mile}$$

Solution

Now, we apply Kron's reduction to eliminate the neutral and obtain a 3 × 3 impedance matrix.

$$Z'_{aag} = \left(Z_{aag} - \frac{\left(z_{ang}\right)^2}{z_{nng}} \right)$$

$$= \left(0.2572 + j1.3770 - \frac{(0.0954 + j0.7523)^2}{0.8013 + j1.6032} \right) = 0.3244 + j1.0632$$

and

$$Z'_{\mathrm{abg}} = \left(Z_{\mathrm{abg}} - \frac{Z_{\mathrm{ang}} Z_{nbg}}{Z_{nng}} \right)$$

$$= \left(0.0954 + j0.8515 - \frac{(0.0954 + j0.7523)(0.0954 + j0.7865)}{0.8013 + j1.6032} \right)$$

$$= 0.1674 + j0.5241$$

Solution

Similarly, we can obtain all the other elements to get the following 3×3 matrix:

$$\begin{bmatrix} 0.3244 + j1.0632 & 0.1674 + j0.5214 & 0.1674 + j0.4067 \\ 0.1674 + j0.5214 & 0.3343 + j1.0353 & 0.1697 + j0.4464 \\ 0.1674 + j0.4067 & 0.1697 + j0.4464 & 0.3244 + j1.0632 \end{bmatrix} \Omega/\mathrm{mile}.$$

5. Series Impedance Models of Underground Lines

5.1 Nonconcentric Neutral Cables

- Among the underground cables used, one usually encounters either nonconcentric (separate) neutral conductors (Fig. a) or concentric neutral conductors.
- The equations of Carson's used for the overhead lines can also be used to evaluate the self and mutual impedance elements for phase conductors in nonconcentric neutral cables.
- Distances from centers of the phase conductors and the neutral can be computed for the given cable and neutral sizes.

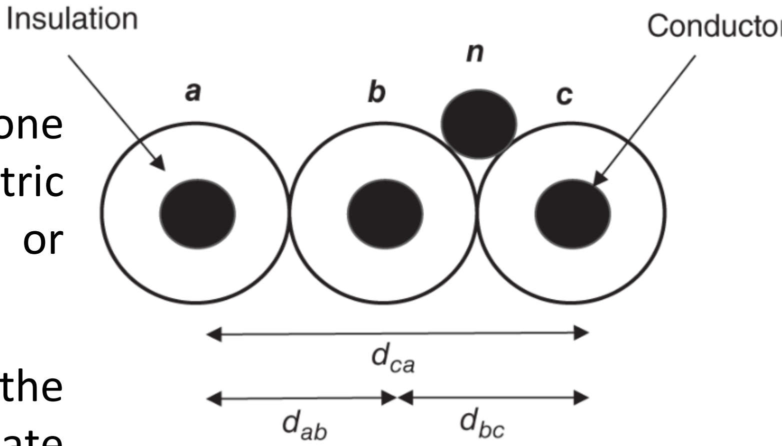


Fig. a: Three-phase underground cable with nonconcentric neutral.

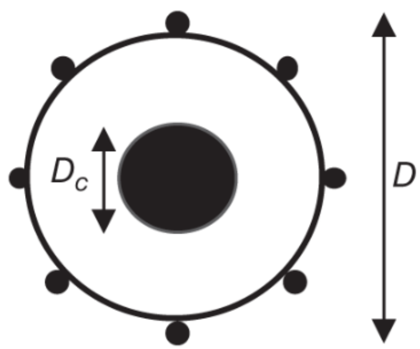


Fig. b: Concentric Neutral

5.2 Concentric Neutral Cables

Concentric neutral cables have several neutral strands at the periphery of the cable as shown in the Figure. These neutral strands help in the distribution of electric field in the cable to reduce stress on the insulation.

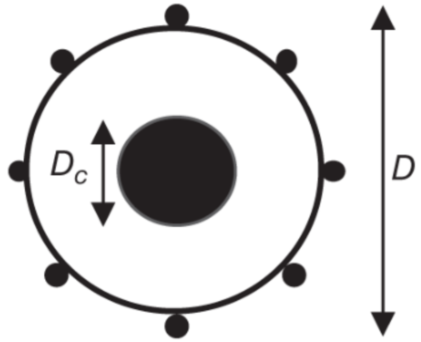


Fig: Single-Phase cable with Concentric Neutral

5.2.1 Single-phase Cable

Terminologies :

- **N** number of neutral strands around the phase conductor
- $\boldsymbol{D_c}$ diameter of the phase conductor
- **D** outer diameter of the cable over the neutral strands

5.2 Concentric Neutral Cables

5.2.1 Single-phase Cable

Terminologies :

- $\boldsymbol{D_s}$ diameter of the individual neutral stands
- $\mathbf{GMR}_p$ geometric mean radius of the phase conductor
- $\mathbf{GMR}_{\boldsymbol{n}}$ geometric mean radius of each neutral strand
- $\boldsymbol{GMR_N}$ geometric mean radius of ensemble of neutral strands of each phase
- $\boldsymbol{D_{an}}$ distance from the center of each neutral strand to the center of the phase conductor
- $\boldsymbol{R_a}$ resistance of the phase conductor in Ω/ mile
- $\boldsymbol{R_n}$resistance of a single neutral strand in Ω/ mile.

5.2.1 Single-phase Cable

- Note that all the distances associated with cable geometry and configuration must be converted to feet (ft). Single subscript for resistance of phase and neutral conductors is used.
- Computation all the information that is needed for computation of line impedance:

$$D_{an} = \frac{D - D_s}{2}$$

$$\mathrm{GMR}_n = e^{-1/4}\frac{D_s}{2} = 0.7788\frac{D_s}{2}$$

5.2.1 Single-phase Cable

- Since the individual neutral stands are solid, the factor $e^{-1/4}$ is used to compute its GMR. This factor accounts for the magnetic field internal to the conductor. In the next step, we compute GMR_N.

$$\mathrm{GMR}_N = \sqrt[N]{N \cdot \mathrm{GMR}_n (D_{an})^{(N-1)}}$$

5.2.1 Single-phase Cable

- Now, we can compute self-impedance of the phase and equivalent neutral conductor and mutual impedance:

$$Z_{aag} = (R_a + 0.00159f) + j\left(0.004657f\log\frac{2160\sqrt{\frac{\rho}{f}}}{GMR_p}\right)\Omega/mile$$

$$Z_{nng} = (R_n/n + 0.00159f) + j\left(0.004657f\log\frac{2160\sqrt{\frac{\rho}{f}}}{GMR_N}\right)\Omega/mile$$

$$Z_{ang} = Z_{nag} = 0.00159f + j\left(0.004657f\log\frac{2160\sqrt{\frac{\rho}{f}}}{D_{an}}\right)\Omega/mile$$

5.2.1 Single-phase Cable

- These impedances give the 2 × 2 matrix of the cable, which is

$$\begin{bmatrix} z_{\mathrm{aag}} & z_{\mathrm{ang}} \\ z_{\mathrm{nag}} & z_{\mathrm{nng}} \end{bmatrix}$$

- Finally, Kron's reduction is used to find the series impedance of the cable with an assumption that the neutrals are grounded.

$$z'_{\mathrm{aag}} = \left(z_{\mathrm{aag}} - \frac{\left(z_{\mathrm{ang}}\right)^2}{z_{nng}} \right)$$

5.2.2 Three-phase Cable

- An example of a three-phase cable with concentric neutrals is provided in the Figure.
- The required quantities can be calculated in a manner similar to the single-phase cable. Distances between the phase conductors are already specified.
- Equivalent distances of neutral strands of one conductor to the other phase conductors can be approximated by distances from the center of the conductor to other conductors. For example, the equivalent distance of phase a neutral strands to phase b conductor is: $D_{n_a} = d_{ab}$

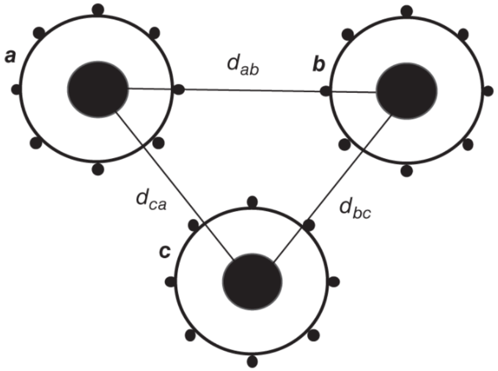


Fig: Schematic of a three-phase cable with concentric neutral conductor

5.2.2 Three-phase Cable

- For computing the impedance matrix of this cable arrangement, we consider three separate neutrals associated with each of the phase cables.
- The self-impedance of each phase, and each neutral ensemble is same as the expression of single-phase cable. (page 23)
- Mutual impedances can be computed between:
  - phase conductors
  - phase conductors and neutrals
  - neutrals of separate phases

5.2.2 Three-phase Cable

- Mutual impedances between phase conductors:

$$Z_{ikg} = Z_{kig} = 0.00159f + j\left(0.004657f\log\frac{2160\sqrt{\frac{\rho}{f}}}{d_{ik}}\right)\Omega/\text{ mile}$$

$$i,k = a,b,c \text{ and } i \neq k$$

- Mutual impedances between phase conductors and neutral strands of the same conductor are same as that of single-phase conductor (i.e., the expression of Z_{ang} , mentioned in slide 23)

5.2.2 Three-phase Cable

- Mutual impedances between phase conductors and neutral strands of another conductor:

$$Z_{in_kg} = Z_{n_kig} = 0.00159f + j\left(0.004657f\log\frac{2160\sqrt{\frac{\rho}{f}}}{d_{ik}}\right)\Omega/\text{ mile}$$

$$i,k = a,b,c\ and\ i \neq k$$

- Mutual impedances between neutral strands of one phase and neutral strands of another phase:

$$Z_{n_in_kg} = Z_{n_kn_ig} = 0.00159f + j\left(0.004657f\log\frac{2160\sqrt{\frac{\rho}{f}}}{d_{ik}}\right)\Omega/\text{ mile}$$

$$i,k = a,b,c \text{ and } i \neq k$$

5.2.2 Three-phase Cable

- With three phase conductors and three equivalent neutral conductors, we get a 6×6 impedance matrix. We can partition this matrix into four partitions as shown below:

$$\begin{bmatrix} Z_{pp} & Z_{pn} \\ Z_{np} & -Z_{nn} \end{bmatrix}$$

Where,

Zpp represents the self and mutual impedances of phase conductors,

Zpn and ***Znp*** represent the mutual impedances between phase and neutral conductors, and

Znn represents self and mutual impedances of neutral conductors.

5.2.2 Three-phase Cable

- Kron's reduction is applied to remove the neutrals and to obtain the impedance matrix for the phase conductors.

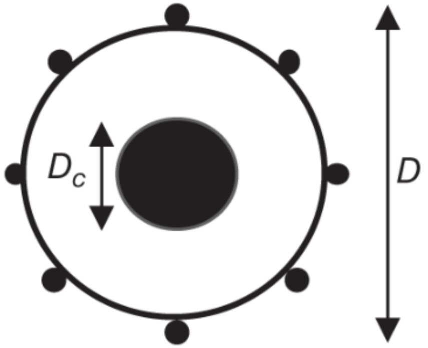
$$[Z'_{pp}] = [Z_{pp}] - [Z_{pn}] \cdot [Z_{nn}]^{-1} \cdot [Z_{np}]$$

Example

Consider a single-phase 15-kV cable with concentric neutral conductors, as shown in Figure below.

The main conductor is 350-kcmil copper with a diameter of 0.661 inch, the outer diameter of the cable is 1.486 inches, and there are 16 concentric neutral conductors of #9 AWG. The GMR of the main conductor is 0.0214 ft, the alternating current (AC) resistance of the cable at 90 °C is 0.064 Ω/1000 ft, and the direct current (DC) resistance at 25 °C of each neutral strand is 0.832 Ω/1000 ft.

Find the 2 × 2 impedance matrix of the cable for the phase and neutral conductors with impedances represented in Ω/mile. Remove the neutral using Kron's reduction to determine the impedance of the cable with ground and neutral effects included.



Solution

First, we compute the distance from the center of the neutral conductors to the center of the main conductor. The outer diameter of the cable is given, but the diameter of #9 AWG neutral wire is needed. A search for data on wires gives 0.1144 inch as the diameter of this wire. Therefore:

$$D_{an} = \frac{D - D_s}{2} = \frac{1.486 - 0.1144}{2} = 0.6858 \text{ inch or } \frac{0.6858}{12} = 0.05715\text{ft}$$

$$\text{GMR}_n = 0.7788\frac{D_s}{2} = 0.7788 \times \frac{0.1144}{2 \times 12} = 0.003712\text{ft}$$

$$\text{GMR}_N = \sqrt[N]{N \cdot \text{GMR}_n (D_{an})^{(N-1)}} = \sqrt[16]{16 \cdot 0.003712(0.05715)^{(16-1)}} = 0.05728\text{ft}$$

$$R_a = 0.064 \times 5.28 = 0.33792\Omega/\text{mile}$$

$$R_n = \frac{0.832 \times 5.28}{16} = 0.27456\Omega/\text{mile}$$

Solution

Then we have:

$$Z_{\text{aag}} = (0.33792 + 0.00159 \times 60) + j\left(0.004657 \times 60\log\frac{2160\sqrt{\frac{100}{60}}}{0.0214}\right)$$
$$= 0.4333 + j1.4292\Omega/\text{mile}$$

$$Z_{\text{nng}} = (0.27456 + 0.00159 \times 60) + j\left(0.004657 \times 60\log\frac{2160\sqrt{\frac{100}{60}}}{0.5728}\right)$$
$$= 0.3699 + j1.3097\Omega/\text{mile}$$

$$Z_{\text{ang}} = Z_{\text{nag}} = 0.00159 \times 60 + j\left(0.004657 \times 60\log\frac{2160\sqrt{\frac{100}{60}}}{0.5715}\right)$$
$$= 0.0954 + j1.3100\Omega/\text{mile}$$

Solution

Therefore, the 2×2 impedance matrix of the cable is

$$\begin{bmatrix} 0.4333 + j1.4292 & 0.0954 + j1.3100 \\ 0.0954 + j1.3100 & 0.3699 + j1.3097 \end{bmatrix} \Omega/\text{ mile}$$

Further,

$$\boldsymbol{Z}'_{\text{aag}} = \left(0.4333 + j1.4292 - \frac{(0.0954 + j1.3100)^2}{0.3699 + j1.3097}\right)$$

$$= 0.5955 + j0.1714\Omega/\text{mile}$$

Thank You!